OPTIMAL CONTROLLER DESIGN FOR INVERTED PENDULUM SYSTEM:A COMPARATIVE STUDY

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ABSTRACT

Inverted pendulum system is a typical model of multivariable, nonlinear, essentially unsteady system, which is perfect experiment equipment not only for pedagogy but for research because many abstract concepts of control theory can be demonstrated by the system-based experiments.The research on such a complex systeminvolves many important theory problems about system control, such as nonlinear problems, robustness ability and tracking problems. The simplest case of this system is the cart- single inverted pendulum system.The main aim is to stabilize the inverted pendulumsuch that the position of the cart on the track is controlled quickly and accurately so that the pendulum is always erected in its inverted position during such movementsby designing optimal controller such as Linear Quadratic Regulator (LQR) controller. In this study, the modelling and simulation for optimal control design of nonlinear inverted pendulum-cart dynamic system using PID controller and pole placement method is also performed.
Keywords:Inverted Pendulum(IP), LQR, PID, Pole Placement.

1. INTRODUCTION

Being an inherently unstable system, the inverted pendulum is among the most complex systems, and is one of the most important classical problems. Due to its importance, this is a choice of dynamic system to analyse its dynamic model and propose a control law.
The Inverted Pendulum is a classical control problem in dynamics and control theory and is widely used as a benchmark for testing control algorithm (PID controller, neural network, fuzzy control, genetic algorithm etc.).The Inverted Pendulum System is Single Input Multiple Output (SIMO) type of system. Here, there are two number of free component i.e. it has 2 degree of freedom. It has one input and the two outputs are position and angle.

2. RELATED WORK

As a linear inverted pendulum system, the mathematical model will be established by the method of mechanical analysis as follows. The Schematic diagram ofthe inverted pendulum is shown in Fig 2.1.Isolation force analysis of the car as shown in Fig 2.2, and isolation force analysis of the rod as shown in Figure 2.3.

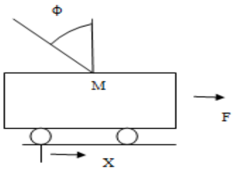


Fig: 2.1

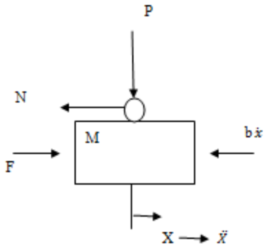


Fig: 2.2

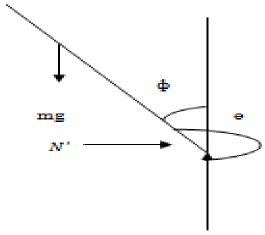


Fig: 2.3

Control input is the force F. The outputs are the angular position of the pendulum θ (theta) and the horizontal position of the cart x. M is the mass of the cart (0.7kgs), m is the mass of the pendulum (0.3kgs) and L is the distance from the pivot to mass centre of the pendulum (0.5m).
By the principles of modern control theory, and then substituted the inverted pendulum system parameter which is designed into the state space equation is given by:

$$\begin{pmatrix} \dot{X} \\ \ddot{X} \\ \dot{\emptyset} \\ \ddot{\emptyset} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+mL^2)b}{P} & \frac{m^2gL^2}{P} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mLb}{P} & \frac{mgL(M+m)}{P} & 0 \end{pmatrix} \begin{pmatrix} X \\ \dot{X} \\ \emptyset \\ \dot{\emptyset} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{I+mL^2}{P} \\ 0 \\ \frac{mL}{P} \end{pmatrix}$$

Here, g is the gravitational constant taken as 9.8kg/m^2

3. CONTROLLER DESIGN

3.1 Design of LQR

A special case of optimal control problem which is of particular importance arises when the objective function is a quadratic function of x and u, and the dynamic equations are linear. The resulting feedback law in this case is known as the linear quadratic regulator (LQR). The performance index is given by:

$$J = \frac{1}{2}\int_{t_0}^{t_f}(x^T Q_x + u^T R_u)dt \ldots\ldots\ldots\ldots\ldots \text{(3.1.1)}$$

Where Q is the symmetric, positive semi-definite state weighting matrix, and R is the symmetric, positive definite control weighting matrix.

$u(t) = -K_x(t)$ …………………… (3.1.2)

Where K is the (m×n) control gain matrix given by

$K = R^{-1}B^T P$ ……………….. (3.1.3)

And P is the unique symmetric, positive semi-definite (n×n) solution of the algebraic Riccatiequation.

$PA + A^T P - PBR^{-1}B^T P + Q = 0$ ……………… (3.1.4)

The design approach used here is the Linear Quadratic Regulator (LQR) method. An LQR controller is designed considering two outputs of inverted pendulum, i.e., cart position and pendulum angle. The four available states considered here are cart position, velocity, pendulum angle and angular velocity. A controller is to be designed such that, when the pendulum is displaced, it eventually returns to zero angle (i.e. the vertical) and the cart should be moved to a new desired position according to the controller.

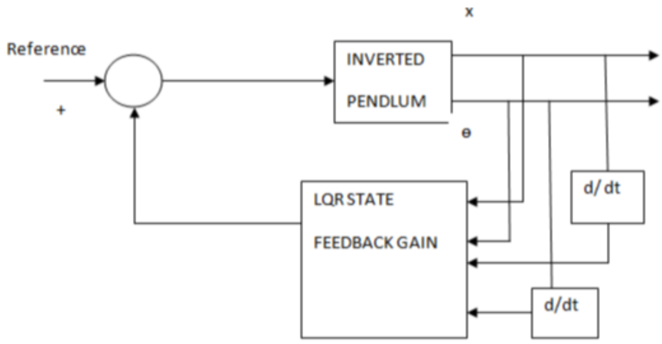


Fig 3.1: Block diagram of LQR controller

For inverted pendulum, firstly,Q and R is chosen as [500 0 0 0;0 1 0 0;0 0 500 0;0 0 0 1] and 1 respectively. Therefore, K[22.3607 18.2401 21.9933 -3.8968].Secondly Q and R is chosen as [1000 0 0 0;0 1 0 0;0 0 1000 0;0 0 0 1] and R=0.01 respectively and K is found to be K= [316.2278 228.0634 239.7219 -80.1712]

3.2 Design of PID Controller

A proportional-integral-derivative controller (PID controller**)** is a control loop feedback mechanism (controller) widely used in industrial control systems. A PID controller calculates an error value as the difference between a measured process variable and a desired set point. The controller attempts to minimize the error byadjusting the process through use of a manipulated variable.To stabilize the inverted pendulum in the upright position and to control the cart at the desired position using the PID control approach, two PID controllers: Angle PID controller and cart PID controller have been designed for the two control loops of the system. The equations of the PID control are given as:

$V_p = K_{pp}e_\theta(t) + K_{ip}\int e_\theta\, dt + K_{dp}\frac{d\theta_\theta}{dt}$ ………………………. (3.2.1)

$V_c = K_{pc}e_x(t) + K_{ic}\int e_x\, dt + K_{dc}\frac{d\theta_x}{dt}$…………………. (3.2.2)

Where $e_\theta(t)$ and $e_x(t)$ are angle error and cart position error, respectively. Since the pendulum angle dynamics and cart position dynamics are coupled to each other, the change in any controller parameters affects both the pendulum angle and cart position, which makes the tuning tedious.

3.3 Pole placement method

It is one of the classic control theories and has an advantage in system control for desired performance. In pole placement method, we assume that state variables are measurable and are available for feedback. If the considered system is completely state controllable, then we can place the poles of the closed loop system in any desired locations by means of state feedback through an appropriate state feedback gain matrix.In the conventional approach to the design of a single-input-single-output control system, we design a controller (compensator) such that the dominant closed-loop poles have a desired damping ratio ζ and an undamped natural frequency ω_n.The requirement is that the system be completely state controllable.

Consider a control system

$\dot{x}$ = Ax + Bu …………………....(3.3.1)

y = Cx + Du…………………… (3.3.2)

Where, x=state vector (n-vector), y=output signal (scalar), u=control signal (scalar), A=n× n constant matrix, B=n × 1 constant matrix, C=1 × n constant matrix,D=constant (scalar).We shall choose the control signal to be

u = −Kx

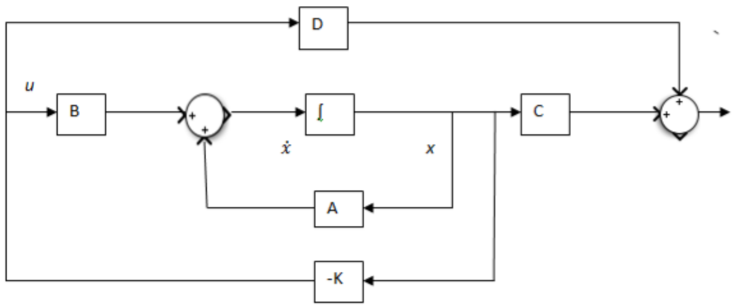


Fig 3.4:Block diagram of Pole Placement method

This means that the control signal u is determined by an instantaneous state. Such a scheme is called state feedback. The 1xn matrix K is called the state feedback gain matrix. We assume that all state variables are available for feedback. In the following analysis we assume that u is unconstrained.The present work, Ackermann's formula method has been used to determine the matrix K.For an arbitrary positive integer n, we have

$K = [0\ 0\ ...\ 0\ 1]\ [B:AB:....: A^{n-1} B]^{-1}\ \Phi(A)$ ……………….(3.3.3)

Equation (3.3.3) is known as Ackermann's formula for the determinationof the state feedback gain matrix K.

3.4. SIMULATION RESULTS

The simulation results using LQR with linear inverted pendulum is shown below:
Fig.3.4.1 and Fig.3.4.2 shows the step response of linear inverted pendulum taking two different values of Q and R respectively as mentioned above.

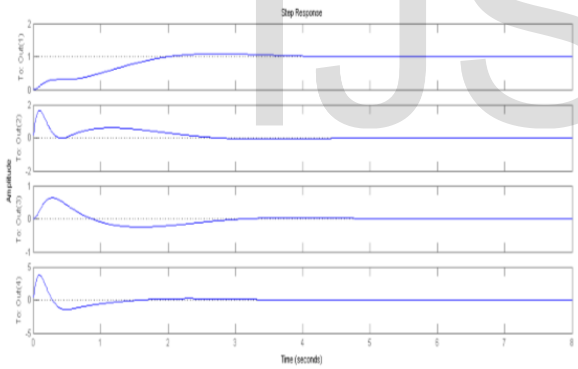


Fig 3.4.1: Step response of Inverted pendulum Using LQR

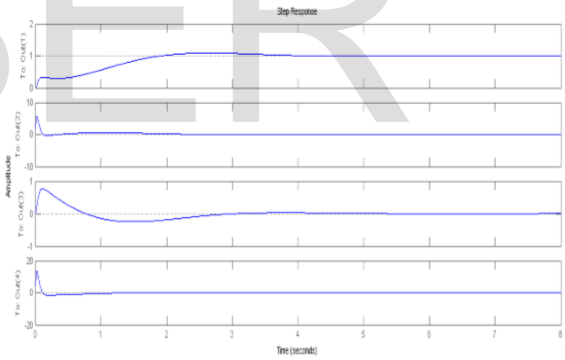


Fig 3.4.2: Step response of Inverted pendulum using LQR

The simulation result for the LQR scheme for the cart position, linear velocity of the Cart, angle of Pendulum, angular velocity is shown in the table below. The speed of reaching the final value depends on choice of Q matrix. From the results obtained, it can be concluded that with the high value of Q matrix, settling time decreases and also the rise time decreases.

Table I. Characteristics of fig.(3.4.1)

Parameters	Settling time(in seconds)	Rise time(in seconds)
x	3.99	1.69
$\dot{x}$	4.66	0
θ	4.85	0
$\dot{\theta}$	3.85	0

Table II. Characteristics of fig. (3.4.2)

Parameters	Settling time(in seconds)	Rise time(in seconds)
x	3.8	1.61
$\dot{x}$	2.28	0
θ	4.61	0
$\dot{\theta}$	1.18	0

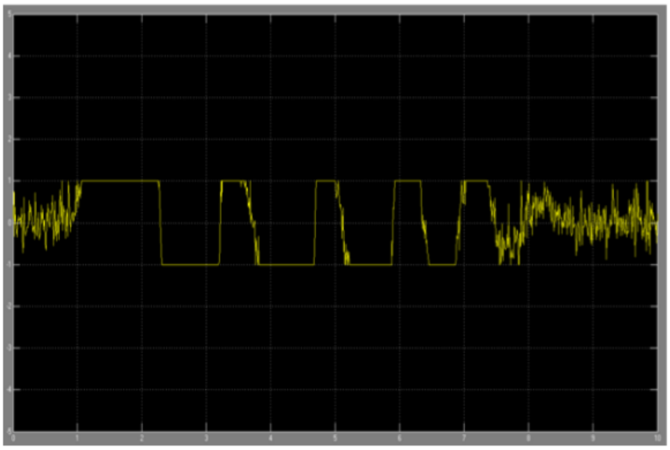


Fig 3.4.3: Force input (u)

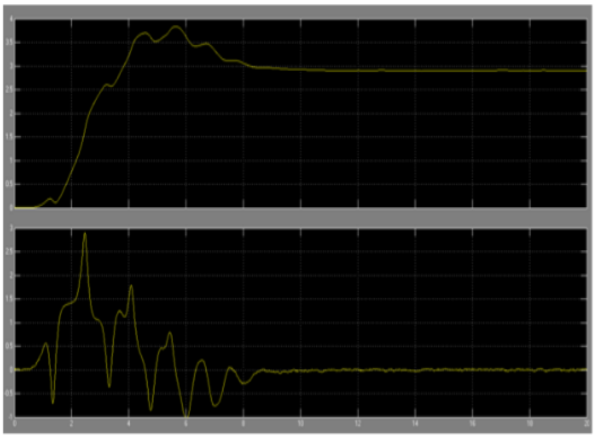


Fig 3.4.4: Plot of $x$and $\dot{x}$ versus time

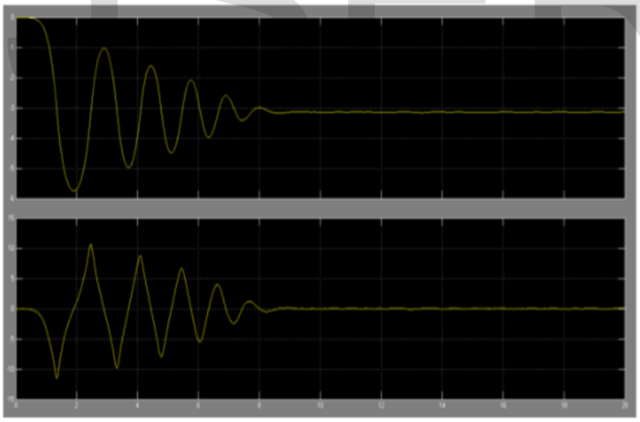


Fig 3.4.5: Plot of θ and $\dot{\theta}$versus time

In the above, fig 3.4.3, fig 3.4.4 and fig 3.4, the simulation of inverted pendulum using LQR controller in Matlab/Simulink is shown.

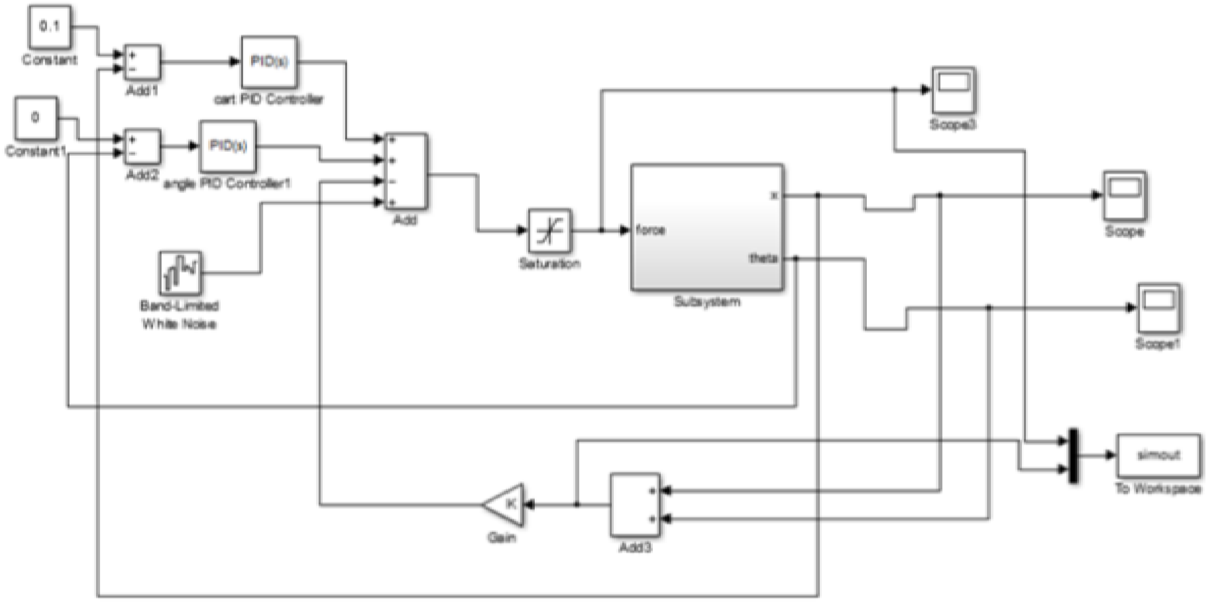
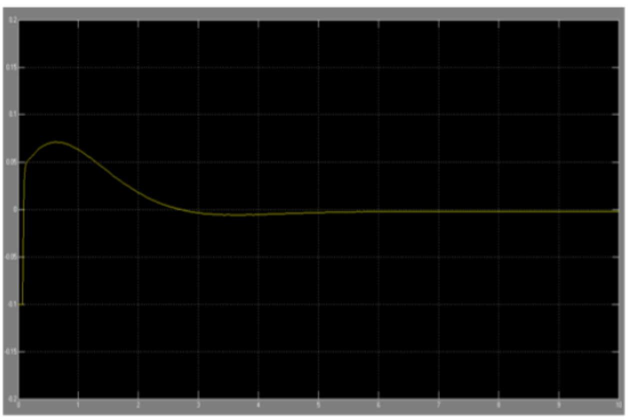


Fig 3.4.6: PID control of inverted pendulum system

From the block diagram of PID controller of Inverted Pendulum, it is seen that the pendulum angledynamics and cart position dynamics are coupled to each other, the change in any controller parameters affects both the pendulum angle and cart position, which makes the tuning tedious.The disturbance input parameters taken in the simulation are: band limited white noise power = 0.0001, sampling time = 0.1, seed= 23341. The tuned PID controller parameters of thiscontrol schemes for cases of with disturbance input is given below:

Table III: PID controller parameters of control schemes for with disturbance input case

Control Scheme	Angle	PID	Control	Cart	PID	Control
	K_{pp}	K_{ip}	K_{dp}	K_{pc}	K_{ic}	K_{dc}
(2PID + LQR)	1	1	1	1.6	-8	6



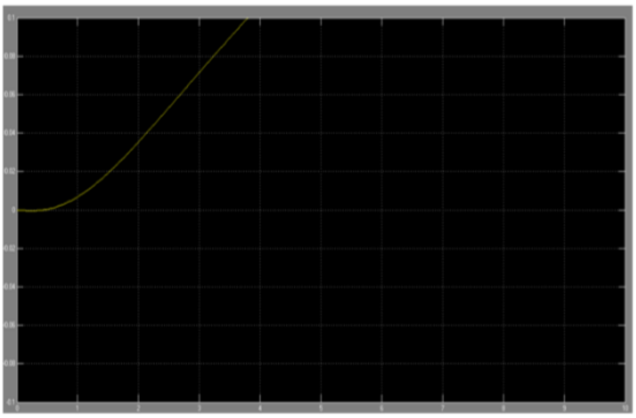


Fig 3.4.7: Input to pendulum

Fig 3.4.8: Plot of x versus time

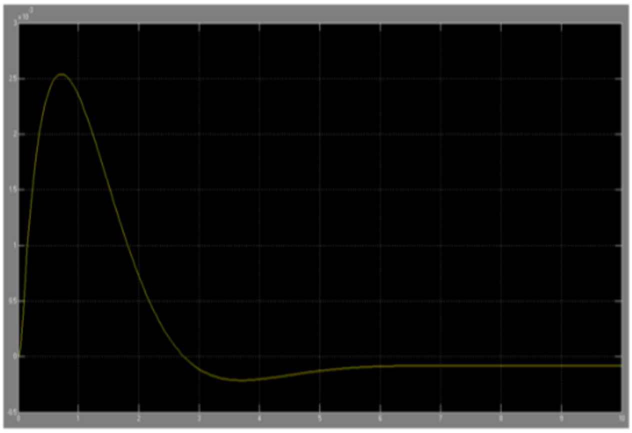


Fig 3.4.9: Plot of θ versus time

Here only pendulum angle θ and cart position x are considered for themeasurement.The reference angle is set to ‘0 radian’, and reference cartposition is set to 0.1 m. The simulation results for both cases are shownin figure. It is observed that the pendulum stabilizes it alsostabilizesupright with minor oscillations for the case of with continuous disturbance input. The Matlab-Simulink models have been developed for simulation and performance analysis ofthe control schemes.

For Pole Placement given the single-input inverted pendulum system and a vector ’ecl’ of desired self-conjugate closed-loop pole locations, MATLAB command ‘place’ computes a gain matrix ’G’ such that the state feedback $u = -Gx$ places the closed-loop poles at the locations of ’ecl’. In other words, the Eigen values of $A - BG$ match the entries of ’ecl’. Table IV shows the settling time and rise time of the system. The G matrix is found to be:

G= [0.2565 0.5822 2.0808 0.7701]

Table IV: Characteristics table

Parameters	Settling Time (in seconds)	Rise time (in seconds)
Position	6.5	2.73
Angle	7.9	0

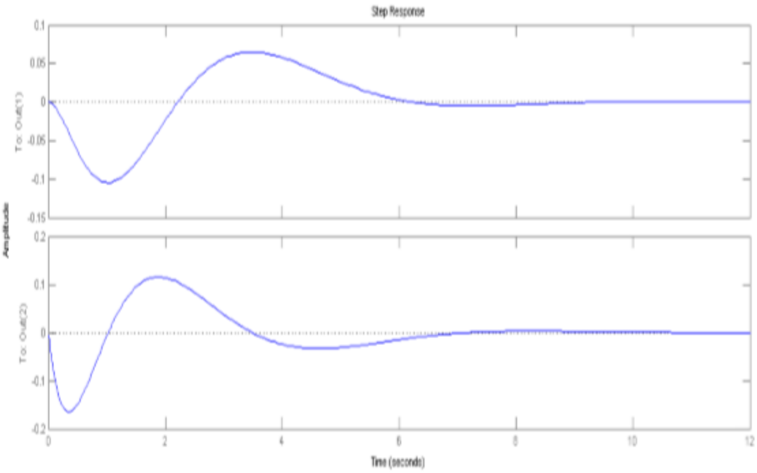


Fig 3.4.10: Pole placement method in Inverted Pendulum

Thus, the study presents a number of control approaches such as LQR, PID and Pole Placement for inverted pendulum system. These design method have been successful in meeting the stabilization goals of the IPS.The results show that LQR method give the better performancecompared to PID controller and Pole Placement by reducing overshoot, settling timeand minimize the rising time.

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